B.M.S COLLEGE FOR WOMEN AUTONOMOUS
BENGALURU – 560004
SEMESTER END EXAMINATION – JANUARY/FEBRUARY 2023

B.Sc MATHEMATICS - I Semester
ALGEBRA –I AND CALCULUS -I
(NEP Scheme 2021-22 onwards F+R)

Course Code: MAT1DSCT01　　　　　　　　　　　　　　　　**QP Code: 1015**
Duration: 2 ½ Hours　　　　　　　　　　　　　　　　　　**Max. Marks: 60**

I. Answer any Six Questions:　　　　　　　　　　　　　　　　(2x 6=12)

1. Find the value of 'k' so that the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & k \\ 3 & -2 & 3 \end{bmatrix}$ is 2 .

2. Define eigen value and eigen vector.

3. Show that the function f(x)= $\begin{cases} \dfrac{x^2-9}{x-3}, \text{for } x \neq 3 \\ 6 \text{ , for } x = 3 \end{cases}$ is continuous at x=3

4. Find the nth derivative of $\log_e(1\text{-}2x)$.

5. State Cauchy's mean value theorem.

6. Evaluate using L'Hospitals rule $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$.

7. If $z = \dfrac{x}{y} + \dfrac{y}{x}$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

8. Find total differential of u= x siny-y sinx.

II. Answer any TWO Questions:　　　　　　　　　　　　　　　(6x2=12)

1. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ by reducing to echelon form.

2. Solve the system of equations x+2y+3z=0, 2x+3y+4z=0, 7x+13y+19z=0.

3. Find eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

III. Answer any Six Questions: (6x6=36)

1. a) Discuss the continuity of f(x) = $\frac{1}{1+e^{-1/x}}$ if x $\neq$0 and f(0) =0 at x=0

   b) Find the nth derivative of $\dfrac{x^2}{(x+2)(2x+3)}$

2. If x=sint and y=cospt, then prove that $(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 - p^2)y_n = 0$

3. Prove that a function which is a continuous in a closed interval is bounded.

4. Verify Lagrage's Mean Value Theorem for f(x)= x²-3x+2 in [-2,3].

5. a) Expand using Maclaurin's series $f(x) = \log_e(secx)$.

   b) Evaluate using L'Hospitals rule $\lim\limits_{x\to0}\left(\dfrac{\tan x - x}{x^2 \tan x}\right)$.

6. a) If u =f (x - y, y – z, z – x), prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$.

   b) If x= rcosθ, y= rsinθ, then find $J = \dfrac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \dfrac{\partial(r,\theta)}{\partial(x,y)}$.

7. State and prove Euler's theorem on homogeneous function.

8. Investigate the extreme values of the function f(x,y)=2x²–xy +y² +7x.

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